Title: Investigating the Use of Concept Mapping as Tools in Mathematics Education

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Abstract. In the actual didactical discussion there is a widespread consensus that mathematics should be experienced by students as a network of interrelated concepts and procedures rather than a collection of isolated rules and facts. This experience may be supported by representing mathematical knowledge graphically in the form of networks.
In this paper, the special graphical representation by concept maps is focused. It is argued, that concept mapping is an especially suited means to experience but also to learn the network character of mathematics. There are pointed out some possible applications of concept mapping in mathematics education, supported with educational studies and classroom experiences. Advantages and limits of these possible uses are discussed.
Further, the paper deals with the question, if concept maps that follow the rules first given by Novak & Govin (1984) are the best suitable graphical network representations in mathematics education, or if modified concept maps or other forms of knowledge maps might be more efficient for this purpose.
It turns out that concept mapping may be an efficient tool to improve mathematics achievement, and that a further development and modification of the traditional concept mapping might provide even more benefits for mathematics education.

Keywords: mathematical networks; connections; concept maps; knowledge maps; mind maps

Investigating the Use of Concept Mapping as Tools in Mathematics Education

Astrid Brinkmann

1 Introduction

Mathematical knowledge has the character of a network, as mathematical objects, i.e. for example concepts, definitions, theorems, proofs, algorithms, rules, theories, are manifold interrelated but also connected with components of the external world. Accordingly, there is a widespread consensus in the actual didactical discussion that mathematics should be experienced by students in its interrelatedness rather than as a collection of isolated rules and facts (see e.g. NCTM Yearbook, 1995, Preface, or NCTM Principles and Standards for School Mathematics, 2000, p. 64). The importance of this notion also becomes apparent in the PISA–Study, where interconnections and common ideas are central elements (OECD, 1999, p. 48).

The network character of mathematics may be experienced but also learned by visualizing graphically structure in mathematics. An especially suited means for this purpose is concept mapping, representing a mathematical network around a topic in a well-structured graphical display. Mwakapenda & Adler (2003) e. g. state, that "concept mapping provides entry into reflecting on … connections".

The special fitting of this technique as a pedagogical tool for mathematics education, especially with regard to build structure, is pointed out in this paper and possible uses of concept maps together with their advantages and limits are discussed, and supported with educational studies and classroom experiences (paragraphs 2 and 3).

Further more there is posed the question, if concept maps provide the most efficient knowledge display with regard to mathematics education. There is presented a study, weighting the use of concept maps against that of mind maps in the students' views, and additionally a study indicating a way for further development of knowledge mapping concerning mathematics (paragraph 4).

2 The adequacy of concept mapping for representing and for learning mathematical networks

Concept maps were first introduced by Novak as a research tool, showing in a special graphical way the concepts related to a given topic together with their interrelations. The method of concept mapping "has been developed specifically to tap into a learner's cognitive structure and to externalise … what the learner already knows" (Novak & Govin, 1984, p. 40), according to Ausubel's statement: "The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly" (Ausubel et al., 1980; Novak, 1998, p. 71). Although the primer intention was to use concept mapping in research, it was found this to be also a useful tool in helping students to "learn how to learn"

(Novak & Govin, 1984; Novak, 1990, 1996). Consequently, concept mapping has been used also as an educational tool, above all in science, whereas experiences in *mathematics* education are rather seldom and not well-documented (Malone & Dekkers, 1984, p. 225; Hasemann & Mansfield, 1995, p. 47; Raymond 1997).

However, both, the structure of a concept map and the technique of concept mapping (Novak & Govin, 1984, p. 33-34; Novak, 1998, p. 227-228[1]) emphasize the adequacy of mathematical issues as topics for concept maps (Brinkmann, 2007, 2003b; see also Brinkmann, 1999, 2001a, 2001b). There is a great agreement between cognitive psychologists, that the internal representation of knowledge resembles well organized and structured networks of ideas (Hiebert & Carpenter, 1992; Hiebert & Lefevre, 1986). Such networks may be modeled by graphs, the vertices showing the interrelated ideas or concepts and the edges representing the existing links between them (Brinkmann, 2008, p. 34-37). Concept maps are special graphs of this kind. They are hierarchically structured, conforming to the general assumption that the cognitive representation of knowledge is hierarchically structured (Tergan, 1986; Jüngst, 1992). Mathematical knowledge may thus be organized in a concept map according to this knowledge's mental representation.

As for mathematics, this "is often depicted as a mighty tree with its roots, trunk, branches, and twigs labeled according to certain sub disciplines." (Davis & Hersh, 1981, p. 18). Similarly, the structure of a concept maps resembles on the whole a tree, only seen from another perspective. Connecting lines from upper concepts to lower concepts along the branches of a concept map are especially suited to represent relations according to mathematics subject systematics. Some essential of these relations are (Brinkmann, 2001b, 2002a):

- different interpretations of the inclusion relation (part-whole link, subset-superset link, subconcept-superconcept link, case distinction link, classification link, characteristic/feature link (i.e. link between a characteristic/feature of a mathematical object and this object)),
- relation of deduction (deduction link, i.e. link between a mathematical object and another deduced from it),
- relation of belonging (belonging link, i.e. for example link between a theorem and a proof of this theorem, link between a problem and its solution).

[1] In brief, Novak (1998, p. 227-228) lists following activities for building a concept map:

1. Identify a focus question that addresses the problem, issues, or knowledge domain you wish to map. Guided by the question, identify 10 or 20 concepts that are pertinent to the question and list these.
2. Rank order the concepts by placing the broadest and most inclusive idea at the top of the map.
3. Work down the list and add more concepts as needed.
4. Begin to build your map by placing the most inclusive, most general concept(s) at the top.
5. Next select the two, three, or four subconcepts to place under each general concept.
6. Connect the concepts by lines. Label the lines with one or a few linking words.
7. Rework the structure of your map, which may include adding, subtracting, or changing superordinate concepts.
8. Look for crosslinks between concepts in different sections of the map and label these lines.
9. Specific examples of concepts can be attached to the concept labels.
10. Concept maps could be made in many different forms for the same set of concepts.

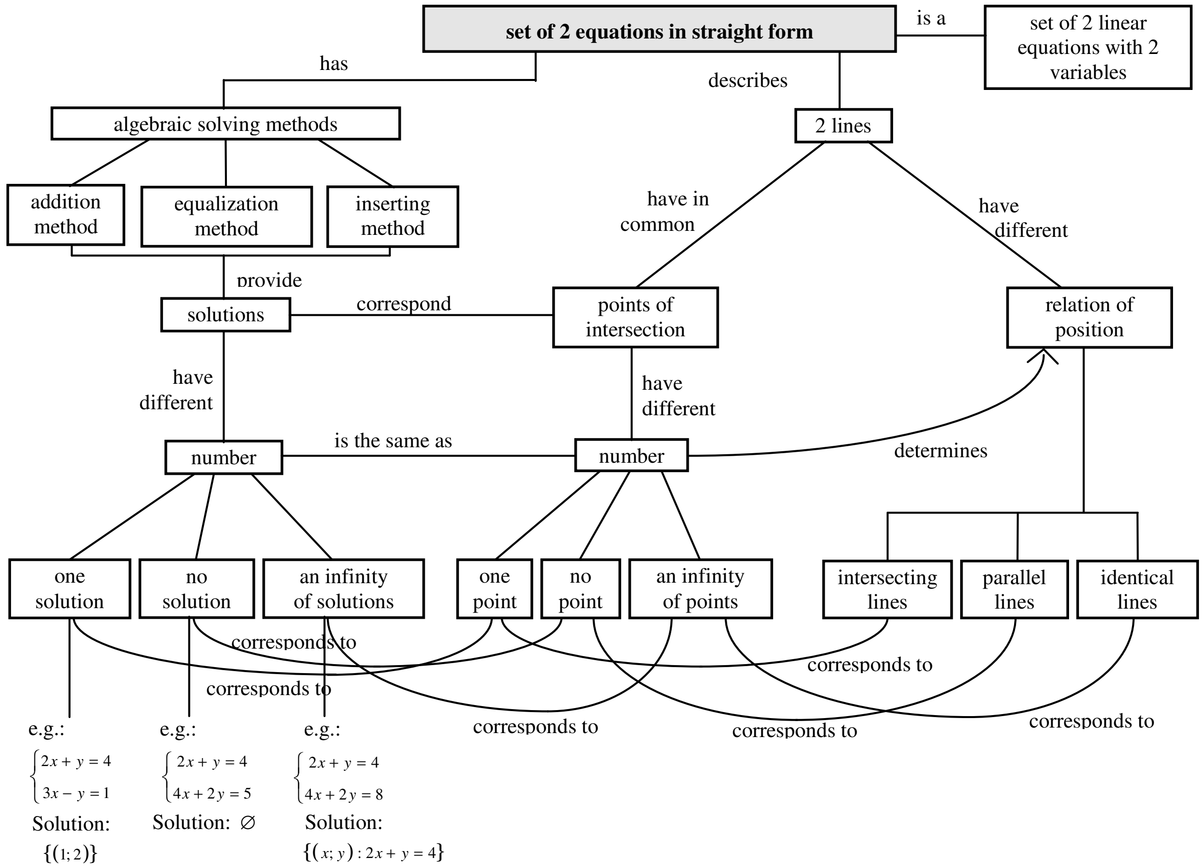

Figure 1. Concept map on the topic of linear equations

In addition, concept maps show also links between concepts of different branches. These cross-links allow particularly the representation of connections between different representations of one and the same mathematical object (for example a geometric representation and an algebraic representation), the so-defined "model links" (Brinkmann, 2001b, 2002a). These links are often used in mathematical problem solving processes in order to get solutions for a mathematical problem using representational change.

The structure of concept maps is thus in accordance with the character of interrelatedness of mathematics: relations between mathematical objects around a topic may be visualized by concept maps in a structured way that corresponds to the structure in mathematics.

3 Uses of concept maps in mathematics education – Studies, classroom experiences and discussion

3.1 Uses of concept mapping in mathematics education

Below there are listed and explained some of the most important uses of concept maps, which might be profitable also in *mathematics* education. Most of them are named by Novak & Govin (1984), Novak (1990, 1996) and Malone & Dekkers (1984), but in view of lacking experiences especially in mathematics classes they are to be seen more likely as thesis. Where possible, they are supported with more recent mathematics education studies, reported in literature or carried out by the author, or with reports of teachers of different German schools about classroom experiences (Brinkmann, 2007; see also 2003b, 2005b).

- Concept maps help to organize information on a topic. Particularly, they aid in organizing and understanding new subject matter and facilitate meaningful learning.

  About it, the concept maps may be preset in the textbook or by the teacher, or constructed by the students themselves. The outcomes of some studies, mostly involving science content, provide evidence that "there is little difference in the effectiveness of teacher-prepared concept maps compared with student-prepared concept maps in improving students' achievement, but when students did prepare their own concept maps, they showed much stronger achievement gains if they themselves were required to supply the key terms necessary to construct the maps" (Hasemann & Mansfield, 1995, p. 47).

  As result of a questioning[2] of the students in four classes/courses, where concept mapping has been used in mathematics lessons, and also as feedback from some teacher colleagues, there is a widespread consensus among students that concept maps can help to organize information, especially if these concept maps are drawn by the students themselves. For the usefulness of concept maps in this point, the degree of complexity of a concept map (defined by the represented number of hierarchy levels, of concepts and of links) seems to play a crucial role: maps with a great degree of complexity seem to be rather confusing than helpful. As a problem it must be seen that a productive degree of complexity is dependent on the individual, or at least on the achievement level of a learning group.

- Concept maps are a powerful tool for identifying students' knowledge structures, especially also misconceptions or alternative conceptions.

  Some first studies pertaining to mathematics indicate the validity of concept maps as a respective research tool; concept maps proved to be a useful device for assessing conceptual knowledge (Laturno, 1994; Williams, 1998).

  Identifying students' conceptual knowledge may help the teacher to plan effective lessons by taking into account what a learner already knows. A student himself gets awareness of his own knowledge organization. Possibly wrong connections in a student's knowledge become visible to the teacher and can be corrected by him.

  In comparison to mind maps, that similarly allow organizing mathematical knowledge around a topic, concept maps are more likely to show students' knowledge structures, as there are intended more linking lines and the description of the represented relations by linking words. But experience based it is very hard for students to find suitable linking words, also if they can describe correctly, in longer terms, the represented relation. Thus some additional discussions of the teacher with the single students might be helpful.

- Concept maps may be used to show growth in students' knowledge (Wilcox & Sahloff, 1998).

  Wilcox & Sahloff (1998) report about a particular strategy of having students construct a concept map at the beginning of a unit of instruction and then again at the end of the unit.

[2] The students were asked if concept maps were helpful for them, and if yes, in which respect. Also for the answer "no" students had to give reasons.

This is not only profitable for the teacher, who learns a lot about how his/her students' knowledge of a content domain grew over time, it may also serve as self-assessment for students: "The two maps were a powerful visual representation to the students themselves of how much they had learned in the unit." Furthermore, Wilcox & Sahloff (1998) report, that the students' concept maps can be a powerful tool for communicating to parents what their child has accomplished.

- Concept maps may serve as a memory aid.

  As a concept map is a graph, a pictorial representation, it may be grasped at once, and due to its unique appearance committed well to one's memory and recalled faster.

- Concept maps may be used for revision of a topic.

  At the end of a topic a concept map can be constructed, as repetition and in order to get a lasting and well organized overview of this topic.

  In classes where concept mapping has been introduced, it could be observed, that some of the students constructed by their own initiative concept maps in order to prepare for written tests at the end of a topic. Higher achievers generally constructed more detailed and comprehensive maps than lower-achievers.

- Concept maps can be used as design of instructional materials.

  Teachers found that concept maps were useful tools for organizing a lecture or an entire curriculum. Moreover, they were not only aided in planning instruction, but also their own understanding of the subject matter was increased (Novak, 1996).

- In my view, it might also be profitable to use concept maps with gaps, with the aim of learning mathematical connections (Brinkmann, 2007, p. 63-65).

  Here, students have to fill in missing concepts and links in an incomplete map. By doing so, students have to be concerned themselves actively and intensively with the contents of the respective topic and their interconnections. Of course, it might occur that there are different possibilities for sensitive entering an empty frame. Teachers thus have to be open for unexpected solutions and should in every case analyze their meaningfulness.

- Such gappy concept maps suit, with certain assumptions, also for testing conceptual mathematical knowledge.

  For this, one should use a concept map that had been discussed in the classes before. In order to avoid that students fill in only elements learnt by heart, the shape of the concept map should be changed. This can be done easily by changing for example the order of the concepts of a hierarchy level.

  I myself have posed a problem of this style in a written test of one of my courses. I have used a concept map to the topic of pairs of simultaneous linear equations, similar to that one shown in figure 1, but in a much more detailed version.[3] In the test, the two concepts "algebraic solving methods" and "2 lines" were put in reverse order, accordingly the

[3] This map is shown on the site http://www.math-edu.de/Concept_Mapping/CM-LGS.pdf in German.

order of concepts in lower hierarchies were changed. In the last concept row most of the frames were empty and only one cross link was drawn. (Figure 2 shows a cutting of this map.)

All the students dealt successfully with this problem, showing that they could meaningfully complete the presented map.

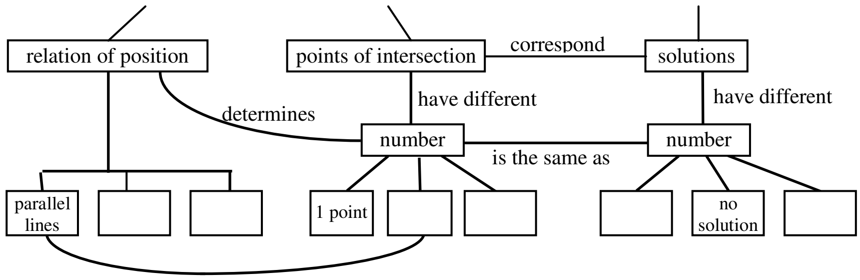

Figure 2. Cutting of a gappy concept map on the topic of linear equations

- In order to be useful, knowledge must be organized so as to facilitate problem-solving ability (see Malone & Dekkers, 1984, p. 222). Concept maps organize knowledge and thus may be useful for problem-solving processes.

  The statement that concept maps may facilitate mathematical problem-solving abilities is not well-founded. For this purpose, I have carried out a study (Brinkmann, 2007, p. 65-67, 2005a) in 4 high school mathematics courses. The students had to work on several mathematical problems, that were unusual for them and that afforded a well-organized conceptual knowledge on the concerned topic. All students could use their textbooks, half of the students could additionally use a concept map representing the needed knowledge in a structured form and constructed by the author. The students had been distributed equally according to their achievement levels into the 2 groups. It turned out, that those students using the concept map were more successful in solving the given problems. Moreover the study showed, that the given concept map was more helpful for normally higher-achievers than lower-achievers. The lower-achievers expressed, that the given concept map was too complex for them, they would have preferred a map showing only an excerpt of it, or the division of the concept map into two maps. Perhaps it is of importance, that the students that participated at this study were not accustomed to the use of concept maps, this representation form was new for them.

  Another study concerning the usefulness of concept maps in problem solving processes was carried out in a course with 28 students that had already experiences with concept mapping. The students had to solve same problems on a mathematical topic, which were not solvable in a routine manner. They were allowed to use a self-made concept map to the corresponding topic. After dealing with the problems they had to assess the usefulness of their map for solving the problems. 25 out of the 28 students answered that they could very well use their map; it helped them for solving all or at least a part of the problems. One student could not profit of his map at all, as the posed problems were too difficult for him, and the remaining two students did not use their maps, as they succeeded in solving the problems without them.

There remain a lot of open questions: what degree of complexity is optimal for students with a certain achievement level, which differences can be observed between students that are accustomed to concept mapping and novices, is it more helpful for problem-solving to use a (comprehensive) map constructed by the teacher or the whole class or to use a self-constructed map, …

- Concept mapping may improve attitudes towards mathematics.

  As by means of concept maps, an individual's mathematical knowledge may gain more structure and clarity the individual's viewpoint on mathematics may become more positive. Furthermore, concept maps enable students through their visualisation to make the sustainable experience, that mathematics is not a collection of isolated rules and facts but a network of ideas in which each idea is connected to several others. The authors of the Curriculum and Evaluation Standards for School Mathematics (NCTM 1989) "contend that the establishment of connections among mathematical concepts enables students to appreciate the power and beauty of the subject" (Hodgson, 1995, p. 13). Thus concept mapping may contribute to a change of an individual's beliefs on mathematics giving them a more positive emotional loading.

3.2 Limitations

It has to be considered that the method of concept mapping generally can be used only if one has got familiar with it. Moreover, when using concept mapping it has to be calculated that it takes some time to construct a concept map.

In spite of their well-structured and ordered contents concept maps may sometimes have a confusing effect. The allowed complexity of a map, so that it is useful, depends on the achievement level of each individual.

When interpreting a concept map, it has to be taken into consideration that the map can in each case only represent a knowledge excerpt; not represented concepts or links might even so be part of the knowledge of the map's author. As well, from the represented interconnections one cannot unambiguously conclude the depth of understanding of the topic.

The organization of concepts on the different hierarchical levels of a map can sometimes be done in different meaningful ways. One has to decide in such a case for one aspect of ordering, and only this aspect becomes visible for others. This has to be taken into account by teachers when assessing their students' maps.

Not all relationships shown in a concept map will signal meaningful learning, because concept links can be learned by rote as well (Novak & Govin, 1984, p. 107). This also might be the case for mathematics education.

Concept maps have a rather rigid structure, thus not allowing adding easily every new idea one might associate to the topic, as this is the case for example by mind maps[4], that

[4] Mind maps are hierarchically structured knowledge displays around a topic. They are produced following the rules given below:

- Place the topic of the mind map in the center of a large sheet of paper.

likewise help organizing information around a topic. But in a mind map, each main branch builds up a complex whole with its sub branches, and connections between the single complexes are as a rule not drawn in order to increase the clarity of the map. Thus, the existing relations to the map topic are probably represented incomplete in a mind map. In contrast, the concepts of a concept map are linked by lines whenever they are related in some way; moreover, every single relationship is described by linking words written on the linking lines. Thus, a concept map provides much more information on a topic than a mind map, but it has, as pointed out above, not got that open structure.

One may ask if the traditional rules for concept mapping as described by Novak & Govin (1984; see also Novak, 1998) are the most suitable with regard to mathematics education. Perhaps somewhat modified concept mapping rules or the use of related methods, such as for example mind mapping, might be more profitable in the case of mathematics teaching and learning. The next paragraph deals with studies according to questions like these.

4 Investigating the most appropriate knowledge maps in mathematics education

The probably best known and most used knowledge mappings are concept mapping and mind mapping, both of which there exist several slightly differing methods. Concept mapping and mind mapping were invented on the basis of different theories and following different aims, but it turns out that both methods seem to be efficient tools for mathematics education (Brinkmann, 2000, 2001c, 2002b, 2003a).

Of course, depending on the pursued goals, teachers have to decide, theory- or experience-led, which of the two methods they particularly want to use in their lesson. But how would students decide for themselves, if they would have the choice? And, do students see one of these two mapping forms as optimal for themselves with respect to certain aims?

The studies presented in the paragraphs 4.1 and 4.2 (see also Brinkmann, 2006, 2007) were carried out in a course of grade 10 (with 13 girls and 11 boys) at a gymnasium in Germany. The students of this course had learned both techniques, concept mapping and mind mapping, with their original rules.

4.1 Concept mapping or mind mapping?

4.1.1 Study design

The students had to answer the questionnaire shown in figure 3; the evaluation was done separately according to the sex.

- From the topic draw a main branch for each of the main ideas linked to the topic. Write keywords denoting the main ideas directly on the lines.
- Starting from the main branches you may draw further lines (sub branches) for secondary ideas (subtopics) and so on. The order follows the principle: from the abstract to the concrete, from the general to the special.
- Use colors when drawing a mind map. Add images, sketches, symbols, such as little arrows, geometric figures, exclamation marks or question marks, as well as self-defined symbols to your mind map.

Mind map or concept map?

1. Which of the two representation forms do you find more sensible for yourself? Why?
2. Are there certain situations where you would rather prefer mind mapping and others where you would rather prefer concept mapping? Describe and give reasons.
3. Which representation form would you choose if you want to organize / to structure in a summarizing way contents to a topic:

    Mind map.
    Why?
    Concept map.
    Why?
    Other representation form.
    Which one?
    Why?

Figure 3. Questionnaire

4.1.2 Results

As for *question 1*, 10 students (8 girls and 2 boys) named mind map, 13 students (5 girls and 8 boys) named concept map and one student named a "symbiosis" of both. As reasons for the preference of a mind map there were listed:

- more easily to construct (6 times),
- allows a more clear view (4 times),
- one can make important things stand out by using colors/pictures/symbols (4 times),
- one can grasp well the whole with one view (1 time)
- a mind map allows better to integrate thoughts and to recall these later (1 time),
- one can better note same personal helps (1 time).

The reasons named for concept map were:

- allows a more clear view (12 times),
- looks more ordered (6 times),
- relations from one to another are easier to comprehend / to recognize (3 times),
- one can show better, which relations there exist between the concepts (2 times),
- one can better learn with it (1 time).

The student, who named a "symbiosis" of both, pointed out as a reason, that this would provide the advantages of both.

When answering *question 2*, 5 girls and 1 boy expressed that they always would prefer mind mapping, 2 girls and 7 boys always would prefer concept maps, and 6 girls and 2 boys would decide situation dependent. The students, which would always use mind maps, respectively, would always use concept maps, named essentially the same reasons as to question 1. Those who would decide situation dependent, named in the case of generally preferring mind maps:

- mind mapping, when I have to construct the map by my own, as this is more easy (2 times),
- mind mapping for things that I understand well, concept mapping for things where I have problems (1 time),
- concept map if there are only few sub-concepts to a topic, as this seems to me more clearly (1 time),
- concept map for problem solving, as this one gives a better overview (1 time),

and in the case of generally preferring concept maps:

- mind mapping if one has to construct by oneself the map, concept mapping if there is presented a teacher-prepared map (3 times),
- mind mapping for a topic, where I do not know exactly about the existing connections, as it is easier to be drawn (1 time),
- mind mapping if one has to include many concepts (1 time),
- mind mapping if there are several sub-concepts to a topic (1 time),
- concept mapping to make more clearly the relationships (1 time),
- concept mapping as an aid for problem solving (1 time).

Regarding *question 3*, nearly all students named the same representation form as under question 1, with the same reasoning. One student named additionally the text form in order to make a summary, because this “is better to learn”. Two students with general preference for concept maps named here the text form, because “there is more sense behind”, respectively an exercise-book containing a collection of rules.

4.2 Maps for summarizing repetition and structuring or as an aid for problem solving processes

When my students had the free choice between concept mapping and mind mapping or some self-made modification of these, in order to summarize and structure their knowledge on a topic, or with the aim to use their map in problem solving processes, they usually constructed a mixture of a mind map and a concept: they typically centered the topic (this is advantageous with regard to space problems), drew 3 to 5 main branches in the style of mind mapping, constructed the complex to each main branch rather in the style of concept maps and described the represented relations only partly by linking words. Advantageously, they used the possibility to give examples, also in giving example problem solutions. (For map examples see Brinkmann, 2007, p. 74-75.)

4.3 Summary and discussion

The study results indicate that students have no definite preference regarding the use of concept mapping versus mind mapping. They weight differently the advantages of the one or the other technique. Students orient themselves by the subjective felt effect of the map: ones feel concept maps as giving a more clear view, others mind maps. Females seem to tend more to mind maps, males to concept maps. More than one third of the students would decide situation based in different ways, taking under consideration the specific advantages and disadvantages of the two mapping methods.

For some students the choice of the representation form depends on if they have to construct a map by their own. Mind maps are easier to be drawn, but concept maps are seen as more profitable by these students. As a consequence, students should become help in completing and improving a self-constructed map such as to be more efficient in learning processes.

In order to summarize contents to a topic in a structured way, nearly all students involved in the study would choose the representational form of a map; learners should thus become familiar with these representational means. However, regarding the aim of organizing mathematical knowledge to a topic or of constructing a map as an aid for problem solving processes, students prefer a typical mixture of mind mapping and concept mapping, trying to construct in a most simple way a most profitable map. Regarding these facts, it might be of benefit to optimize the rules for making knowledge maps, especially with regard to build structure in mathematics or as a help in problem solving processes. In addition, as students show having problems in describing the presented relations, it seems to be quite appropriate to provide them with linking words fitting for the usually used mathematics relations. It might also be helpful to come to the agreement not to describe each sort of links.

5 Outlook

As knowledge mappings can be expected to be efficient tools for building structure, and hence also for facilitating problem-solving ability, an enhanced usage of these methods in mathematics education should result. A further development and modification of the traditional concept mapping might provide even more benefits particularly for mathematics education.

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